MATH2060B: Analysis II: Home Project 3 **30 April 2021 at 5:00pm**

Important Notice:

♣ The answer paper **Must be submitted before 1 May 2021 at 5:00pm.**
♠ The answer paper MUST BE sent to the CU Blackboard.
✠ The answer paper **Must include your name and student ID.**

Answer ALL Questions

1. **(15 points)**

   Let $f(x) := \sum_{n=1}^{\infty} x^n(1-x)$. Let $D := \{x \in \mathbb{R} : \ f(x) \text{ is convergent}\}$.

   (a) Find D.

   (b) Does $f(x)$ converge uniformly on D?

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2. **(15 points)**

   Let g be a real analytic function on $\mathbb{R}$.

   (a) Suppose that there is $\delta > 0$ such that $g(x) = 0$ for all $x \in (-\delta, \delta)$. Show that $g \equiv 0$ on $\mathbb{R}$.
   (Hint: Consider the set $\{r > 0 : \ g \equiv 0 \text{ on } (-r, r)\}$.)

   (b) Show that if $\int_a^b |g(x)|dx = 0$ for some $a < b$, then $g(x) \equiv 0$ on $\mathbb{R}$.

3. **(20 points)**

   For each $a \in \mathbb{R}$, put

$$a^+ = \begin{cases} a, & \text{if } a > 0, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad a^- = \begin{cases} -a, & \text{if } a < 0, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Suppose that the series $\sum a_n$ is conditionally convergent, that is, the series $\sum a_n$ is convergent but $\sum |a_n| = \infty$. Show that $\sum a_n^+ = \sum a_n^- = \infty$.

   (b) Consider $a_n := \frac{(-1)^{n+1}}{n}$ for $n = 1, 2, ...$ Show that there is a bijection σ on $\mathbb{Z}+$ such that $\liminf s_n = 0$ and $\limsup s_n = 1$, where $s_n := \sum_{k=1}^n a_{\sigma(k)}$.

$*\,*\,*$ **END OF PAPER** $*\,*\,*$